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**

**SUBJECT: FURTHER MATHEMATICS CLASS: SS2**

**FIRST TERM SCHEME OF WORK**

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| **WEEK** | **TOPIC** |
| 1 | Finding quadratic equation with given sum and product of roots, conditions for equal roots, real roots and no root |
| 2 | Tangents and Normals to Curves |
| 3 | Polynomials ;definition, basic operations + , x , - , ;-- |
| 4 | Polynomials ( Continued) factorization |
| 5 | Cubic Equation , roots of cubic equations |
| 6 | Review and Test |
| 7 | Logical Reasoning ; fundamental issues and definitions and theorem proving |
| 8 | Trigonometric Function , six trig functions of angles of any magnitude ( sine, cosine,tangent,secant, cosecant, cotangent) |
| 9 | Relationship between graph of trigonometric ratios such as sin x and sin 2x, graphs of y= a sin (bx) + c , y = a cos (bx) + c , y = a tan (bx) + c |
| 10 | Graphs of inverse by ratio and equation of simpletrgonometric identities |
| 11 | Revision |

**REFERENCES**

* Further Mathematics Project 1 by TuttuhAdegun
* Further Mathematics Project 2 by TuttuhAdegun
* Additional Mathematics by Godman

**WEEK 1**

**TOPIC: SOLUTION TO QUADRATIC EQUATION**

**FINDING QUADRATIC EQUATION GIVEN SUM AND PRODUCT OF ROOTS CONDITION FOR EQUAL ROOTS, REAL ROOTS AND NO ROOT**

**We recall that if ax2 + bx + c = 0, where a, a and c are constants such that a ≠ 0, then,**

x = or x =

**Suppose we represent these distinct roots by α and β; thus:**

α =

and

β

**We may also put D = b2 – 4ac, so that**

α=

β =

**Sum of roots**

α + β = +

=

=

**Products of roots**

αβ =

˸αβ = b2 – D

4a2

= b2 – (b2 – 4ac)

4a2

= 4ac

4a2

=

Hence, if ax2 + bx + c = 0, where a, b and c are constants andα≠ 0 then α + β= ,

αβ =

x2 + x– 42 = 0

then (x – 6) (x – 7) = 0

**Hence the roots of the equation are 6 and -7. In general, if a quadratic equation factorizes into**

(x – α) (x - β) = 0

then α and β must be the roots of that equation.

The general quadratic equation ax2 + bx + c = 0 can also be written as:

x2 + …(1)

**If the roots of the equation are α and β then the above equation can be written as:**

(x –α) (x – β) = 0

x2 – (α – β) x + αβ = 0 ---(2

By comparing coefficients in equations (1) and (2)

-(α + β) =

: α + β =

andαβ =

The above consideration gives rise to two problems:

(a) Given a quadratic equation, we can find the sum and product of the roots.

(b) Given the roots, we can formulate the corresponding quadratic equation.

The quadratic equation whose roots are α and β is

x2 – (α + β) x + α β = 0

**Find the sum and product of the roots of each of the following quadratic equations:**

(a) 2x2 + 3x – 1 = 0

(b) 3x2 – 5x – 2 = 0

(c) x2 – 4x – 3 = 0

(d) ½ x2 – 3x – 1 = 0

**Solution**

(a) 2x2 + 3x – 1 = 0

a = 2; b = 3; c = -1

Let α and β be the roots of the equation, then

α + β=

α β =

(b) 3x2 – 5x – 2 = 0

a = 3; b = -5; c = -2

Let α and β be the root of the equation, then

α + β =

α β =

(c) x2 – 4x – 3 = 0

a = 1; b = 4; c = -3

Let α and β be the root of the equation, then

α + β =

α β =

(d) ½ x2 – 3x – 1 = 0

a = ½, b = -3, c = -1

Let α and β be the root of the equation, then

α + β =

α β = = -2

**Find the quadratic equation whose roots are:**

(a) 3 and -2 (b) ½ and 5

(c) -1 and 8 (d)¾ and ½

**Solution**

The quadratic equation whose roots are α and β is x2 – (α + β) x +α β = 0.

(a) α + β = 3 – 2 = 1, α β = 3 (-2) = -6

: The quadratic equation whose roots are 3 and -2 is x2 – x – 6 = 0.

(b) α β = α β =

:The quadratic equation whose roots are

x2–

or 2x2 – 11x + 5 = 0

(c) α+ β = 7, α β = -8

:α β = 7,α β = -8

:The quadratic equation whose roots are -1 and 8 is x2 – 7x – 8 = 0.

(b) α+ β = α β =

:The quadratic equation whose roots are ¾ and ½ is

x2–

or 8x2 – 10x + 3 = 0

**Symmetric Properties of Roots**

of ax2 + bx + c = 0, then

α + β = α β =

Certain relations involving α and β can also be determined from α + β and α β even when we do not knowα and β distinctively. Such relations are usually said to be symmetric.

They are symmetric in the sense that if α and β are interchanged, either the relation remains the same or is multiplied by -1.

If α≠ β, determine whether or not each of the following is symmetric:

(a) α + β (b) αβ

(c) α2 β2 (d) α2– β2

(e) 3α +2β (f) α2 β2

**Solution**

(a) α+ β =β + α

: α + β is symmetric

(b)αβ = βα

: αβ is symmetric

(c) α2 β2= α2 β2

: α2 β2 is symmetric

(d) α2 – β2= -(α2 – β2)

: α2 – β2is symmetric

(e) 3α + 2β≠ 3β + 2αsince α ≠ β

:3α + 2β is not symmetric

(f) α2+ β2 = β2+α2

:α2 + β2is symmetric

If α and β are the roots of 3x2 – 4x – 1 = 0, find the value of:

(a) α+ β (b) αβ

(c) α2 β2 (d)

(e) (f) α3β3

(g) α–β (h)

**Solution**

a = 3; b = -4; c = -1

(a) α + β =

(b) αβ =

(c) α2 β2 = (α + β)2 - 2αβ

=

(d) ==

(e) = α2β2 =

αβ

(f) α3β3 = (α+β) (α2+β2 – αβ)

= (α+β) (α2+β)2-3αβ)]

=

=

(g) We know that

(α – β)2 = α2+β2 - 2αβ

= (αβ)2 - 4αβ

(α-β) = (αβ)2 - 4αβ

=

=

=

(h) =

=

=

=

2

=

The Graph of y = ax2 + bx + c (a ≠ 0) is called a parabola and has two shapes depending on whether a > 0 or a < 0.

Q

(a) a> 0 (b)

a< 0

P

When a > 0, the lowest point on the graph is called the minimum point, and it occurs when

x =

Also, the line when

a> 0

(a) x (b) a < 0

x

x = x =

**Nature of Roots**

We recall that the solution of

ax2 + bc + c = 0

is x = , where D = b2 – 4ac

Three restrictions can be placed on the value of D.

(a) D > 0

(b) D < 0

(c) D = 0

When D > 0

The roots of the equation are real and distinct. The graph of y = ax2 + bx + c crosses the x – axis at two points.

(a) a> 0 (b) a < 0

D > 0 D > 0

x = α1 x = β1 x = α2 x = β2

If in addition D is a perfect square, the roots are rational, but if D is not a perfect square, the roots are irrational and are always in conjugate pairs.

When D < 0

The roots are not real. They are said to be imaginary as is not a real number. The graph of

y = ax2 + bx + c does not cross the x – axis in this case.

(a) a> 0 a – axis

D < 0 (b) a < 0

D < 0

x – axis

When D = 0

The roots are real and equal. They are said to be coincidental. The graph touches the x – axis at

x =

x =

(a) a> 0 a < 0

D < 0 D = 0

x =

Since D enables us to determine the position of the graph of y = ax2 + bx + c relative to the x – axis, it is called a discriminant.

Determine the nature of roots of the following quadratic equations:

(i) x2 – 3x – 2 = 0

(ii) x2 – 6x + 9 = 0

(iii) 2x2 – 2x + 5 = 0

**Solution**

(i) a = 1; b = -3; c = -2

D = b2 – 4ac

= 9 + 8

= 17<0

Hence the roots of the equation are real and distinct.

(ii) x2 – 2x + 9 = 0

a = 1; b = -6; c = 9

D = b2 – 4ac

= 36 – 36

= 0

Hence the roots are real and equal.

(iii) 2x2 – 2x + 5 = 0

a = 2; b = -2; c = 5

D = b2 – 4ac

= 4 – 40

= -36

Hence the roots are imaginary.

**Evaluation**

1. Find the quadratic equation where roots are

(a) 3 and -2 (b) ¾ and ½

**General Evaluation**

(1) If α and β are the roots of 3x2 – 4x – 1 = 10, find the value of:

(a) α2 + β2 (b) (c)

(d) α3 + β3 (e) α – β

(2) Find the sum and product of roots of these equation

(a) 2x2 + 3x – 1 = 0 (b) 3x2 – 5x – 2 = 0

**Reading assignment**

New Further Maths Project 2 page 8, 9, 10, 11

**Weekend Assignment**

(1) Determine the nature of roots of x2 – 3x – 2 = 0

(a) Real (b) Imaginary (c) Equal (d) Coincidental

(2) If α ≠ β which of the following is not symmetric

(a) αβ = βα (b) α + β = β + α (c) 3α + 2β = 3β + 2α

(d) α2 + β2 = β2 + α2

If α and β are the roots of 2x2 – 7x – 3 = 0, find:

(3)αβ2 + α2

(a)

(4)

(a)

(5)

(a)

**Theory**

(1) Find the constants p, q and r such that 3x2 – 12x + 16 = p (x + q)2 + r

(2) If α and β are the roots of x2 – 10x + 2 = 0, find α3 – β3.

**WEEK 2**

**TOPIC: Tangents and Normal to Curves**

For any curve, is the gradient function. At any point on the curve, at that point, gives the gradient of the tangent at the point. The derivation of y with respect to x at x = x1 is denoted.

x = x1

Recall that the equation of the line of gradient m through (x1, y1) is y – y1 = m(x – x1).

From this equation, we can easily obtain the equation of the tangent.

The straight line, perpendicular to the tangent at the point of contact of the tangent to the curve is called the **Normal** to the curve.

If m1 is the gradient of the Normal, and m is the gradient of the tangent at the point of contact of the tangent to the curve, then

m1 =

So at the point (x1, y1) the equation of the normal is y – y1 = (x – x1)

Find the equation of the tangent and the normal to the curve y = 2x3 – x2 + 3x + 1 at the point x = 1.

**Solution**

Given that y = 2x3 – x2 + 3x + 1

6x2 – 2x + 3

x = x1= 6 – 2 + 3 = 7

If m is the gradient of the tangent at x = 1, then m = 7.

At the point x = 1; y = 2 – 1 + 3 + 1 = 5.

The equation of the tangent at the point x = 1 is

y – 5 = 1(x – 1)

y – 5 7x– 2

If m1 is the gradient of the normal at x = 1, then

m1 =

Hence, the equation of the normal at x = 1, is y – 5 = (x – 1)

7(y – 5) = -1 (x – 1)

7y – 35 = -x + 1

7y + x – 36 = 0

Find the equation of the tangent to the curve x2y + y2x + 3x – 13 = 0 at the point (1, 2)

**Solution**

x2y + y2x + 3x – 13 = 0

x2+ 2xy + 3y2x y2 + 3 = 0

(x2 + 3y2x) + 2xy + y3 + 3 = 0

(x2 + 3y2x) = -2xy – y3 – 3

-2xy – y3 -3

x2 + 3y2x

If m is the gradient of the tangent at the point (1, 2) then

m = x = 1, y = 2

= =

The equation of the tangent is therefore

y – 2 = (x – 1)

13 (y – 2) = -15 (x – 1)

13y – 26 = -15x + 15

13y + 15x – 41 = 0

**A curve is defined by**

f(x) = x3 – 6x3 – 15x – 1. Find

(i) the derivation f(x) with respect to x;

(ii) the gradient of the curve at the point where x = 1;

(iii) the maximum and the minimum points.

**Find the maximum and minimum points of the curve** y = x3– x2 – 5x and sketch the curve.

**A curve passes through the point (1, 0) and its gradient at any point p(x, y) is 3x2 – 1.**

(i) Find the equation of the curve;

(ii) Sketch the curve, indicating all turning points and the point of intersection with the axes.

Sketch the curve y = x3 – 6x2 + 9x.

Find:

(i)the equation of the tangent to the curve y = x3 + x2 – 8x + 2 at the point A(1, -4);

(ii) the coordinate of the point where the tangent meet the x – axis.

**Evaluation**

**Find the equation of the normal to each of the following curve**

(4) y = (2x – 3) (x + 2) at x = 1

**General Evaluation**

**Find the equation of the tangent to each of the following curve at the given points**

(1) y = x2 – 3x – 4 at x = 1

(2) y = x3 + 2x2 – 3x + 1 at x = -1

(3) y = 1 – 2x + 5x2 – x3 at x = 3

Find the equation of the normal to each of the following curve

(5) y = 6 – 2x + 3x2 – 2x3 at x = 0

**Reading Assignment**

New Further Maths Project 2 page 143, 144, 169

**Weekend Assignment**

A curve y = 4x3 – 2x2 + 7x + 5 at point x = 3, find the

(1) Gradient of its tangent

(a) 12 (b) 108 (c) 103

(d) 115

(2) Gradient of its normal

(a)

(3) Equation of the tangent

(a)y = 108x – 116 (b) y = 103x – 193 (c) y = 115x - 90

(d) y = 12x - 309

(4) Equation of the normal

(a) 103y + x – 11 = 0 (b) 103x + y – 116 = 0 (c) 116x + x – 119 = 0

(d) 116x + y – 103 = 0

(5) Find the gradient of this curve y = 2x2 – 5x + 8 at point x = 1

(a) 1 (b) -1 (c) 2 (d) 3

**Theory**

(1) Find the gradient of the tangent and the normal of the curve y = x3 – 6x2 – 15x – 1 at point x = 1

(2) A curve passes through the point (1, 0) and its gradient at any point p(x, y) is 3x2 – 1, find the equation of the curve.

**WEEK 3**

**TOPIC: Polynomials**

Consider the expressions formed from the sum of integral non negative powers of a variable x taken together with some numerical constants. Such expressions are called **Polynomials.**

The general polynomial takes the form anXn + an1Xn1 + … a2X2 + a1X + a0 where a11 an-1…a0(a0 ≠0) are numerical constants.

The numerical constants an’ an1…. a2, a1 are called coefficients of Xn, Xn1, … X2, X respectively while a0 is called the constant term of the polynomial.

The highest power of the variable is n and is called the degree of the polynomial. Let us designate the general polynomial by p(x). Thus:

P(x) =anXn + an-1 Xn-1 + … + a2X2 + a1X + a0.

The following are examples of polynomials:

(a) P1(x) = 3x2 – 2x + 4

(b) P2(x) = 3x4 – 2x2 + x – 1

(c) P3(x) = x + 1

(d) P1(x) = 2x3 + x - 3

The following are not polynomials:

(a) f1(x) = (x2 + 2x – 3)

(b) f1(x) = 3x2 – 4x2 + 2x – 1

2x + 3

(c) f1(x) = (2x – 3)1

**Equality of Polynomials**

The polynomial p(x) = a11X11 + an-1 Xn-1 + … + a2X2 + a1X + a0 is said to be equal to the polynomial.

Q(x) =bnXn + bn-1X-n1 + … b2X2 + b1 X + b0 provided

an = bn’ an 1 = bn 1 … a2 = b2’ a1 = b1, a0 = b0

**Addition and Subtraction of Polynomials**

Let P(x) = anXn + an-1 Xn-1 + … + a2X2 + a1X + a0

Q(x) = bnXn + bn-1X-n1 + … b2X2 + b1 X + b0 then,

P(x) + Q(x) = (an + bn)Xn + (an-1 + bn-1) Xn1 + … + (a2 + b2) X2 + (a1 + b1) X + a0 + b0.

Also,

P(x) - Q(x) = (an-bn) Xn- (an-1- bn-1) Xn1- … + (a2- b2) X2+ (a1- b1) X + a0- b0.

Given that P1(x) = 7x3 – 4x2 + 3x + 4;

P2(x) = 5x2 + 6x + 1 and P3(x) = 4x3 + 2x – 3.

Find:

(a) P1(x) + P2(x)

(b) P1(x) + P3(x)

(c) P1(x) – P2(x)

(d) P3 (x) – P2(x)

(e) P1(x) + P2(x) + P3(x)

**Solution**

(a) P1(x) + P2(x)

= (7x3 – 4x2 + 3x + 4) + (5x2 + 6x + 1)

= 7x3 + (-4x2 + 5x2) + (3x + 6x) + (4 + 1)

= 7x3 + x2 + 9x + 5

(b) P1 (x) + P3(x)

(7x3 – 4x2 + 3x + 4) + (4x3 + 2x – 3)

= (7x3 + 4x3) + (-4x2( + (3x + 2x) + (4-3)

= 11x3 – 4x2 + 5x + 1

(c) P1 (x) – P2 (x)

= (7x3 – 4x2 + 3x + 4) + (5x2 + 6x + 1)

= 7x3 + (-4x3 – 5x2) + (3x – 6x) + (4 – 1)

= 7x3 – 9x2 – 3x + 3

(d) P3 (x) – P2 (x)

= (4x3 + 2x – 3) – (5x2 + 6x + 1)

= (4x3 – 5x2 + (2x – 6x) + (-3 – 1)

= 4x3 – 5x2 – 4x – 4

(e)P1 (x) + P2 (x) + P3(x)

= (7x3 – 4x2 + 3x + 4) + (5x2 + 6x + 1) + (4x3 + 2x – 3)

= (7x3 + 4x3) + (-4x2 + 5x2) + (3x + 6x + 2x) + (4 + 1 – 3)

= 11x3 + x2 + 11x + 2

Given that P1(x) = 2x3 + 4x2 – x + 1

P2(x) = 3x4 + x3 – 2x2 + x – 3 and

P3(x) = 4x3 + 2x – 4

Find:

(a) 2P1(x) + P2(x)

(b) 3P2(x) + 2P3(x)

(c) 3P1(x) – 3P3(x)

(d) P3(x) + 2P1(x) – 3P2(x)

**Solution**

(a) 2P1(x) + P2(x)

= 2(2x3 + 4x2 – x + 1) + (3x4 + x3 – 2x2 + x – 3)

= (4x3 + 8x2 – 2x + 2) + (3x4 + x3 – 2x2 + x – 3)

= 3x-1 + 5x3 + 6x2 – x – 1

(b)3P2(x) + 2P3(x)

= 3(3x4 + x3 – 2x2 + x – 3) + 2 (4x3 + 2x – 4)

= 9x4 + 3x3 – 6x2 + 3x – 9 + 8x3 + 4x – 8

= 9x4 + 11x3 – 6x2 + 7x – 17

(c) 3P1(x) – 3P3(x)

= 3(2x3 + 4x2 – x + 1) -3 (4x3 + 2x – 4)

= 6x3 + 12x2 – 3x + 3 – 12x3 – 6x + 12

= -6x3 + 12x2 – 9x + 15

(d) P3(x) + 2P1(x) – 3P2(x)

= (4x3 + 2x – 4) + 2 (2x3 + 4x2 – x + 1) – 3(3x4 + x3 – 2x2 + x – 3)

= 4x3 + 2x – 4 + 4x3 + 8x2 – 2x + 2 – 9x4 – 3x3 + 6x2 – 3x + 9

: P3(x) + 2P1(x) – 3P2(x) = -9x4 + 5x3 + 14x2 – 3x + 7

The value of p(x) at x = a is denoted by p(a) and is obtained by substituting a for x in the polynomial.

**Example**

Giventhat p(x)= 2x3 + 5x2 – 9x – 18, find;

(a) P(1)

(b) P(-1)

(c) P(2)

(d) P(0)

**Solution**

(a) P(1) = 2(1)3 + 5(1)2 – 9(1) – 18

= 2 + 5 – 9 – 18

= -20

(b) P(-1) = 2(-1)3 + 5 (-1)2 -9(-1) – 18

= -2 + 5 + 9 – 18

= -6

(c) P(2) = 2(2)3 + 5(2)2 – 9(2) – 18

= 16 + 20 – 18 – 18

= 0

(d) P(0) = 2(0)3 + 5(0)2 – 9(0) – 18

= -18

**Multiplication of polynomials**

The product of two polynomials of degrees ***m*** and ***n*** is another polynomial of degree ***m + n.***

***Example***

Given that P1(x) = 2x2 + 5x + 6 and

P2(x) = 3x2 – 2x + 1, find P1P2

**Solution**

***Method 1***

P1 x P2

= (2x2 + 5x + 6)x (3x2 – 2x + 1)

= 2x2(3x2 - 2x + 1) + 5x(3x2 – 2x + 1) + 6(3x2 – 2x + 1)

= 6x4 – 4x3 + 2x2 + 15x3 – 10x2 + 5x + 18x2 – 12x + 6

= 6x4 +(4x3 + 15x3) +(2x2 - 10x2 + 18x2)+ 5x - 12x + 6

= 6x4 + 11x3 + 10x2 – 7x + 6

**Method 2**

2x2 + 5x + 6

3x2 – 2x + 1

2x2 + 5x + 6

-4x3 – 10x2 – 12x

6x4 + 15x3 + 18x2

6x4 + 11x3 + 10x2 – 7x + 6

**Method 2 is usually called Long Multiplication method.**

**Given that** P1(x) = 4x3 – 2x2 + 3x – 1 and P2(x) = 3x3 – 4 find P1(x) x P2(x)

**Solution**

**Method 1**

P1P2 = (3x2 – 4) (4x3 – 2x2 – 3x – 1)

= 3x2 (4x3 – 2x3 + 3x – 1)

-4(4x3 – 2x2 + 3x – 1)

= 12x5 – 6x4 + 9x3 – 3x2 – 16x3 + 8x2 – 12x + 4

= 12x5 – 6x4 – 7x3 + 5x2 – 12x + 4

**Method 2**

4x3 – 2x2 – 3x – 1

3x2 - 4

-16x3+8x2 – 12x + 4

12x5 – 6x4 – 9x3 + 3x2

12x5 – 6x4 – 7x3 + 5x2 – 12x + 4

**Division of Polynomials**

A polynomial of degree *n* can be divided by another polynomial of degree *m* if *n ≥ m.*

Divide the polynomial P(x) = 3x2 -2x + 4 by the polynomial P(x) = x + 2

**Solution**

Since P1 is being divided by P2 it is called the dividend while P2 is called the divisor. The result of division of P1 by P2 is called the quotient and whatever is left after division is called the remainder.

The procedure of division of P1 by P2 can best be demonstrated step by step as follows:

**Step 1**

Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

3x

x + 2 3x2 – 2x + 4

**Step 2**

Multiply each of the terms of the divisor by the quotient.

3x

x + 2 3x2 – 2x + 4

3x2 + 6x

**Step 3**

Subtract the product obtained in step 2 from the first two terms of the dividend and add the next term of the dividend.

3x

x + 2 3x2 – 2x + 4

3x2 + 6x

-8x + 4

**Step 4**

Using -8x + 4 as a new dividend repeat step 1, 2 and 3.

3x – 8(c)

x + 2 3x2 – 2x + 4 (b)

(a) 3x2 + 6x

-8x + 4

-8x – 16

20 (d)

**Note that:**

(a) x + 2 is the divisor;

(b) 3x2 – 2x + 4 is the dividend;

(c) 3x - 8is the quotient

(d) 20 is the remainder

The expression 3x2 – 2x + 4 can be written as:

3x2 – 2x + 4 = (x + 2) (3x – 8) + 20

Dividend Divisor Quotient Remainder

In general P(x) = D(x) x Q(x) + R

P(x) = Dividend

D(x) = Divisor

Q(x) = Quotient

R = Remainder

Divide 4x3 + 6x2 – 2x + y by 2x – 3 and hence find the quotient and the remainder

**Solution**

2x2 + 6x + 8

2x – 3 4x3 + 6x2 – 2x + 7

4x3 + 6x2

12x2 – 2x

12x2 – 18x

16x + 7

16x – 24

31

Quotient = 2x2 + 6x + 8

Remainder = 31

Find the quotient and remainder when 2x4 – 3x3 + x2 – 4x + 5 is divided by x2 + 3x + 1

**Solution**

2x2 – 9x + 26

x2 + 3x + 1 2x4 – 3x3 + x2 – 4x + 5

2x4 – 6x3 + 2x2

-9x3 – x2 – 4x

-9x3 – 27x2 – 9x

26x2 + 5x + 5

26x2 + 78x + 26

-73x – 21

Hence, Quotient = 2x2 – 9x + 26

Remainder = -73x – 21

Find the quotient and remainder when x3 + 8 is divided by x2 – 2x + 4

**Solution**

x + 2

x2 – 2x + 4 x3 +8

x3 – 2x2 + 4x

2x2 + 4x + 8

2x2 + 4x + 8

0

Quotient = x + 2

Remainder = 0

Hence, x3 – 2x2 + 4x is a factor of x3 + 8.

**The Remainder Theorem**

The Long Division Method.Although a little cumbersome enables us to find, not only the quotient, but the remainder as well. Suppose we are given the polynomial f(x) = 2x3 – 3x2 + 4x – 1 and we wish to find the remainders when f(x) is divided by:

(a) x – 1

(b) x – 2

(c) x – 3

By using the long division method we have:

2x2 – x + 3

x– 1 2x2 – 3x2+ 4x – 1

2x2 – 2x2

-x2 + 4x

-x2 + x

3x – 1

3x – 3

2

So when f(x) is divided by x – 1, the remainder is 2.

2x2 – x + 6

x – 2 2x3 – 3x2 + 4x – 1

2x3 – 4x2

-x2 + 4x

-x2 + 2x

6x – 1

6x – 12

11

So when f(x) is divided by x – 2, the remainder is 11.

2x2 – 3x + 13

x – 3 2x2 – 3x2 + 4x – 1

2x2 – 6x2

3x2 + 4x

-x2 + 9x

13x – 1

13x – 39

38

So when f(x) is divided by x – 3, the remainder is 38.

Now, find:

a. f(1)

b. f(2)

c. f(3)

d. f(1) = 2 – 3 + 4 – 1 = 2

e. f(2) = 16 – 12 + 8 – 1 = 11

f. f(3) = 54 – 27 + 12 – 1 = 38

Compare the answers in each of the following pairs

i. (a) and (d)

ii. (b) and (e)

iii. (c) and (f)

What do you notice?

This is a remarkable result and it is not a mere coincidence.

Let us try another one.

Given that H(x) = x3 + 2x3 – 4x + 3, find the remainder when H(x) is divided by

(a) x + 1

(b) x + 2

(c) x + 3

x2 + x - 5

(a) x + 1 x3 + 2x3 – 4x + 3

x3 + x2

x2 – 4x

x2 + x

-5x + 3

-5x – 5

8

When H(x) is divided by x + 1, the remainder is 8.

x2 -4

(b) x + 2 x3 + 2x3 – 4x + 3

x3 + 2x2

-4x + 3

-4 – 8

11

So, when H(x) is divided by x + 2, the remainder is 11.

x2 + x - 1

(c) x + 3 x3 + 2x3 – 4x + 3

x3 + 3x2

x2 – 4x

- x2 + 3x

-x + 3

-x – 3

6

When H(x) is divided by x + 3, the remainder is 6. Now, find:

(d) H(-1)

(e) H(-2)

(f) H(-3)

(d) H(-1) = -1 + 2 + 4 + 3 = 8

(e) H(-2) = -8 + 8 + 8 + 3 = 11

(f) H(-3) = -27 + 18 + 12 + 3 = 6

Compare again the answers in each of the following pairs:

(i) (a) and (d)

(ii) (b) and (e)

(iii) (c) and (f)

What again do you notice?

(a) When H(x) is divided by x + 1 the remainder is H(-1)

(b) When H(x) is divided by x + 2 the remainder is H(-2)

(c) When H(x) is divided by x + 3 the remainder is H(-3)

So we can safely conclude that if H(x) is divided by x – a the remainder is H(a) and this forms the basis of what is called the remainder theorem.

**Theorem**

If the polynomial f(x) is divided by x – a, the remainder is f(a).

***Proof***

The polynomial function f(x) can be written as:

f(x) = (x – a) Q(x) + R …(1)

where x – a is the divisor, Q(x) the quotient and R the remainder.

Put x = a in (1)

f(a) = (a – a) Q(a) + R

f(a) = R

The theorem whose proof we have just established, is called the **RemainderTheorem**. In general, if f(x) is divided by ax + b then the remainder is

f

A special case of the remainder theorem is when f(x) leaves no remainder when it is divided by x – a. We therefore say that x – a is a factor of f(x). The modified theorem is called, **Factor Theorem** and it states:

If f(a) = 0 then x – a is a factor of f(x).

**Example**

Given that f(x) = 3x3 – 4x2 + 2x + 3, find the remainder when f(x) is divided by x – 1.

**Solution**

Let R be the remainder. Then using the remainder theorem, R = f(1)

f(1) = 3(1)3 – 4(1)2 + 2(1) +3

= 3 – 4 + 2 + 3

= 4

Find the remainder when

f(x) = (x + 3)(x – 2)(x + 2) is divided by x + 1.

**Solution**

Let R be the remainder when f(x) is divided by (x + 1) then R = f(-1)

f(-1) = (-1 + 3)(-1 -2)(-1 + 2)

= (2)(-3)(+1)

= -6

Hence the remainder is -6

Find the remainder when f(x) = 2x3 + 3x2 – 4x + 1 is divided by 2x – 1. What conclusion can you draw?

**Solution**

Let R be the remainder when f(x) is divided by 2x – 1 then R = f

f = 2 f3 + 3 f3– 4 f + 1

= ¼ + ¾ - 2 + 1

= 0

Hence2x – 1 is a factor of f(x).

**Evaluation**

1. If g(x) is the quotient when f(x) = 2X3  + 3x2 -2x +1 is divided by x+2 find the remainder when g(x) is divided by x-1

**General Evaluation**

If f(x) = 6x3 + 13x2 + 2x – 5,

(a) show that f(-1) = 0 (b) find the factor of f(x)

(1) When the polynomial f(x) = x4 + px3 + x2 + qx + 1 is divided by x2 + 3x + 2 quotient is x2 – 1 and the remainder is 5x + 3. Find the value of constants p and q.

(2) If (x + 1) is a factor of the polynomial f(x) = x3 + kx2 + 3x + 10. Find the value of the constant k, and factorise the polynomial completely.

**Reading assignment**

New Further Maths Project 1 pages71 – 75 Exercise 6b Q1, 8, 22 and 26

**Weekend Assignment**

1. Given that f(x) = x5 + 4x4 – 6x2 + 2x + 2, find (-1)

(a) 2 (b) -3 (c) -4 (d) -2

(2) Determine the values of p and q if(x – 1) and (x – 2) are factors of 2x3 + px – 4 + q

(a) p = 12, q = 13 (b) p = -13, p = -12 (c) p = -13, q = 12

(d) p = 10, q = 8

(3) Find zero of the polynomial p(x) = x3 + 4x2 + x – 6

(a) x = -1, 2, 3 (b) x = -1, -3 (c) x = 1, -2, -3 (d) x = 1, 2, -3

(4) If f(x) = 6x3 + 13x2 + 2x – 5, find the factors of f(x)

(a) (x – 1) (2x – 1) (3x – 5) (b) (x + 1) (2x – 1) (3x + 5) (c) (x + 1) (2x + 1) (3x – 5)

(d) (x + 1) (1 – 2x) (5 – 3x)

(5) If (2x + 1) is a factor of the polynomial f(x) = 2x3 – 8x + x2 + k, find the value of the constant k. (a) -2 (b) -3 (c) -4 (d) 3

**Theory**

(1) Find the values of the constant p, q and r such that, when the polynomial

f(x) = x3 + px2 + qx + r is divided by (x + 2), (x – 1) and (x – 3), the remainder are respectively -48, 0 and 2. Hence factorise f(x) completely.

(2) If x – 2 is a factor of f(x) = x3 – x2 + px + q and f(x) leaves a remainder of 12 when it is divided by (x – 3), find

(a) the values of the constant p and q

(b) the three values of c for which x3 – x2 + px + q = 0

**WEEK 4**

**TOPIC: polynomial (continued)**

**CONTENT:**

**Factorization of polynomial**

Show that x - 1 is a factor off(x) = 2x3 + 3x2 – 5x – 6

**Solution**

f(-1) = 2(-1)3 + 3(-1)2 – 5(-1) – 6

= -2 + 3 + 5 – 6

Hence x + 1 is a factor of f(x)

Show that x + 1 is a factor of f(x) = x3 + 2x2 – 5x – 6

Hence factorise f(x) completely.

**Solution**

f(-1) =(-1)3 + 2(-1)2 – 5(-1) – 6

= -1 + 2 + 5 – 6 = 0

: x+1 is a factor of f(x)

Using Long Division:

x2 + x - 5

(a) x + 1 x3 + 2x2 – 5x - 6

x3 + x2

x2 – 5x

x2 + x

-6x - 6

-6x – 6

0

x2 + x – 6 = x2 + 3x2 – 2x – 6

= x(x + 3) -2 (x + 3)

= (x + 3)(x – 2)

Hence f(x) = (x + 1)(x + 3) (x – 2)

Factiorizef(x) = x3 + 7x2 – 14x – 8 completely.

**Solution**

In a complete factorised form, f(x) can be written in the form:

f(x) = (x ± p)(x ± r) …(1)

The first term of the expansion of (1) is x3 while the last term of the expansion is ±pqr. Hence, p. q and r must be factors of -8. We can therefore try:

x ± 1

x ± 2

x ± 4

x ± 8

f(-1) = -1 – 7 – 14 – 8 = -30 ≠ 0

: x + r is not a factor of f(x)

f(1) = 1 – 7 + 14 – 8 = 0

: x – 1 is a factor of f(x).

A similar procedure can be used for x ± 2, x ± 4, x ± 8 to find the other two factors. But a less cumbersome procedure is to use long division method once a factor of f(x) is obtained.

x2 – 6x + 8

x – 1 x2 – 7x2 + 14x – 8

x2 –x2

-6x2 + 14x

-6x2+ 6x

8x – 8

8x – 8

0

Now, x2 – 6x + 8 = x2 – 4x – 2x + 8

= (x – 2)(x – 4)

Hence, f(x) = (x – 1)(x – 2)(x – 4)

What must be subtracted from f(x) = x2+2x2-3x +5 so that it will be exactly divisible by x – 2? Hence, find that polynomial H(x) which is exactly divisible by x – 2 and factorise it completely.

**Solution**

f(x) = x2 + 2x2 - 3x + 5

f(2) = 8 + 8 – 6 + 5

= 15

:When f(x) is divided by x – 2 the remainder is 15. Hence 15 must be subtracted from f(x) to make it exactly divisible by x – 2.

Let the new polynomial which is exactly divisible by x – 2 be H(x), then:

H(x) = f(x) – 15

= x2 + 2x2 - 3x + 5– 15

= x2 + 2x2 - 3x –10

Using long division

x2 + 4x +5

x– 2 x2 + 2x2 - 3x – 10

x2 + 2x2

4x2 – 3x

4x2 – 8x

5x – 10

5x – 10

0

But x2 + 4x + 5 is not factorizable, hence H(x) = (x – 2)(x2 + 4x + 5).

Given that f(x – 1) = x2 + 3x –1, find f(3).

Put x + 1 = 3

x = 2

: f(3) = (2)2 + 3(2) – 1

= 4 + 6 – 1

= 9

(1) Given that p1(x) = 5x3 + 3x2 – 2x + 6, p2(x) = x3 + 4x2 – 3x + 1 and p3(x) 2x3 – 3x + 2, find

(i) (p2 – p3) p1 (ii)p2(p1 + p3)

(2) Divide (i) 4x3 + 3x2 – 2x + 1 by x2 + 2x – 1 (ii) x4 + 2x + 3 by x2 – 1 to obtain the

remainder and quotient.

**Evaluation**

Find the quadratic equation whose roots are (i) 3 & -2 (ii) -1 & 8 (iii) ¾ & ½

**General Evaluation**

(1) When x2 + bx + 2 is divided by x + 3 the remainder is 5. Find the value of b.

(2) If 2x2 – (b – 4) x – 4 (b + 2) = 0, has equal roots, find the possible values of b.

(3) Factorise completely x3 + 5x2 – 3x + 1

(4) Solve the following pair of equation simultaneously 4x – 3y = 17, 3x2 - 2y2 + x – 4y = 73

**Reading Assignment**

Further Maths 1 pages 66 – 69 Exercise 6a Q5, 9 and 10

**Weekend Assignment**

(1) Find the remainder when 2x3 – 4x2 + x – 3 is divided by x + 3

(a) 84 (b) -86 (c) -64 (d) 76

(2) Factorise x4 – 1 completely (a) (x + 2) (x – 3) (b) (x + 1) (x – 1) (x2 + 1)

(c) (x + 1) (x + 2) (x – 3) (d) (x2 + 1) (x + 1) (x – 2)

(3) Given that p1(x) = 2x4 + 3x3 – x2 + 2x – 3 and p1(x) = 3x3 + 2x + 2. Find 3p1(x) – 3p1(x)

(a) 6x4 – 3x2 – 15 (b) 5x4 – 3x4 + 2x2 – x + 3 (c) 6x4 = 9x3 – 15

(d) 6x4 + 18x2 + 6x2

(4) Given that p(x) = x3 + 4x2 – 3x + 1, find p( ½ )

(a)

(5) Given that p(x) = ax2 + bx + 1, p( ½ ) = ½ and p(-2) = 23, determine the values of a and b

(a) a = -3, b = 4 (b) a = 2, b = -3 (c) a = 4, b = -3 (d) a = 3, b = -3

**Theory**

(1) Find the quotient and remainder when 2x4 – 3x3 + x2 – 4x + 5 is divided by x2 + 3x + 1

(2) If p1 = 3x3 + 2x2 – x + 2, p2 = 2x2 + x – 6 and p3 = x3 + 3x2 + 2x – 4, find

(i) p3(p1 + p2) (ii) p2 + p3 – 3p1 (iii) p2 x (p3 + p1)

**WEEK 5**

**TOPIC: Cubic equations and their factorization , graphs of cubic equations**

Polynomials of degree three have the general form y = ax3 + bx2 + cx + d(a ≠ 0). The curve is usually called a **cubical parabola**.

A cubical parabola has two shapes depending on whether a > 0 or a < 0.

a> 0

a< 0

Sketch each of the following curves represented by the following functions:

(a) y = x3 + 2x2 – 5x – 6

(b) y = 12 + 4x – 3x2 – x3

**Solution**

(a) y = x3 + 2x2 – 5x – 6

Using the factor theorem and long division the expression can be factorized as:

y = (x – 2)(x + 1)(x + 3)

The zeros of the polynomial are therefore x = 2, x = -1 and x = 3, hence the x – intercepts are (2, 0), (-1, 0) and (-3, 0).

The y – intercept is (0, -6).

Next, we shall consider the behaviour of the function at different intervals along the x – axis. This will enable us to see whether the curve is above or below the axis.

Mark the x – intercepts on the x – axis.

x–axis

(-3, 0) (-1, 0) (2, 0)

The intervals we shall consider are:

(a) x< -3

(b) -3 < x < -1

(c) -1 < x < 2

(d) x> 2

We shall examine the signs (+ve or –ve) in each of the intervals.

x = -4 is in the intervals x < -3

f(-4) = -18 < 0

Hence the part of the graph in the interval x < -3 is below the axis.

x = -2 is in the interval -3 < x <-1

f(-2) = 4 > 0

Hence the part of the graph in the interval -3 < x < 2 is above the x – axis x = 0 is in the interval -1 < x < 2

f(0) = -6 < 0.

Hence the part of the graph in the interval -1 < x < 2 is below the x – axis.

x = 3 is in the interval x > 2

f(3) = 24 > 0

Hence the part of the graph in the interval is above the x – axis.

The intercept on the axis coupled with the behaviour of the function at different intervals on the x – axis will enable us to get the shape of the curve.

y

x

(-3, 0)(-1, 0) (2, 0)

(0, 6)

(b) y = 12 + 4x – 3x2 – x3

Using the factor theorem and long division method

y = (2 – x)(2 + x)(3 + x)

The zeros of the polynomial are x = 2, x = -2 and x = -3, hence the x – intercepts are (2, 0), (-2, 0) and (-3, 0).

The y – intercept is (0, 12).

Mark the x – intercept on the axis.

x – axis

(-3, 0) (-1, 0) (2, 0)

The intervals we shall consider are:

(a) x< -3

(b) -3 < x < -1

(c) -1 < x < 2

(d) x> 2

x = -4 is in the interval x < -3

f(-4) = 12 < 0,

hence the part of the graph within this interval is above the x – axis.

x = -2.5 is in the interval -3 < x < -2

f(-2.5) =

hence the part of the graph within this interval -3 < x < -2 is above the axis.

x = 0 is in the interval -2 < x < 2

f(0) = 12 > 0,

hence the part of the graph within this interval -2< x <2 is above the axis.

x = 3 is in the interval x > 2

f(3) = -30< 0,

hence the part of the graph within the interval r > 2 is below the x – axis.

y

(0, 12)

(-3, 0) (-2, 0) (2, 0)

**Evaluation**

Sketch the curve of these equations

(a) y = x3 – 6x2 + 11x – 6 (b) y = x3 + 3x2 – 6x – 8

**General Evaluation**

(1) Factorise the following completely

(a) x3 + 10x2 + 23x + 14 (b) x4 - 1

**Weekend Assignment**

Given that a cubic equation x3 + 2x2 – 19x – 20 = 0 has 4 as one its roots, find the

(1) Second root (a) -1 (b) 1 (c) 2 (d) 3

(2) Third root (a) 5 (b) -8 (c) 3 (d) 2  
(3) Sum of the second and third roots (a) -4 (b) 6 (c) 4 (d) -6

(4) Product of the second and third roots (a) 6 (b) 5 (c) -6 (d) -5

(5) Find the zeros of x2 – 1 (a) 2 or -2 (b) 1 or 2 (c) -1 or 1 (d) 1 or -2

**Theory**

(1) If (x + 1) is a factor of f(x) = x3 + kx2 + 3x + 10, find the value of the constant k.

(2) Factorise f(x) completely.

**WEEK 6**

Review

WEEK 1 FINDING QUADRATIC EQUATIONS WITH GIVEN SUM AND PRODUCT OF ROOTS

WEEK 2 TANGENT AND NORMAL TO CURVES

WEEK 3 POLYNOMIAL

WEEK 4 POLYNOMIAL ( CONTINUED)

WEEK 5 CUBIC EQUATIONS

**WEEK 7**

**TOPIC: Logical reasoning**

**Statements**

A statement in a logical context is a declaration, verbal or written that is either true or false but not both.

A true statement is said to have a truth value **T,** while a false statement is said to have a truth value **F.**

**Example 1**

The following are statements:

(a) Nigeria is an African country.

(b) The earth is conical in shape.

(c) If I run I shall not late.

(d) Japanese are hard working people.

**Example 2**

The following are not statements in the logical context.

(a) Who is he?

(b) What a lovely man!

(c) Take the pencil away.

(d) If I think of my family.

In general, questions, exclamations, commands and expressions of feelings which cannot be assigned a truth value T or F are not statements in the logical context.

By convention, we shall use letter *P*.*q.r,…* to denote statements.

**Negation**

Given a statement P, the negation ofP, written ̴P is the statement; “it is false that P” or “nor P”.  
If P is true, ̴P is false and if P is false ̴P is true. In other words, if P has the truth value T then ̴P has the truth value F and if P has the truth value F then ̴P has the truth value T.

The relationship between P and ̴P can be summarizer in the following table.

**Table 1**

|  |  |
| --- | --- |
| P | ̴P |
| T | F |
| F | T |

Table 1 is called a truth table.

If P is a statement: “Nigeria is a rich country” then ̴P is the statement: “it is false that Nigeria is a rich country” or in a more reasonable English “Nigeria is not a rich country”.

Let *q* be the statement “some lawyers are honest people” then ̴*q* is the statement” it is false that some lawyers are honest people”. In a more reasonable English, we can also write ̴*q* as some lawyers are not honest people.

Let P be the statement 3 + 4 = 8 then ̴P is the statement 3 + 4 ≠ 8.

Let q be the statement X + 1 ≥ 4 then ̴*q* is the statement X + 1 ≥ 4.

Let P be the statement “The set of numbers 2, 4, 6, 8, … is a set of even numbers” then ̴P is the statement “The set of numbers 2, 4, 6, 8, … is a set of odd numbers.

1. State which of the following are statements in the logical context:

(a) Caesar was a great leader.

(b) Stop talking to the boys.

(c) Decide whether you are going to the club’s meeting now.

(d) Oh Mansa Musa, you are wonderful!

(e) The Broking House in Ibadan, is a magnificent building.

2. State which of the following are statements in the logical context:

(a) As old as Methuselah.

(b) The set of numbers 3, 5 and 4 is not a Pythagorean triplet.

(c) Is he a serious teacher at all?

(d) If 6 is an odd number, then 3 + 5 = 10.

3. Write the negation of each of the following statements:

(a) He is a handsome man.

(b) It is very cold in Siberia.

(c) It is very hot in tropics.

(d) The sky is blue.

4. Write the negation of each of the following statements:

(a) The party leader will win the election.

(b) The football captain scored the first goal.

(c) Short cuts are dangerous.

(d) Honest men are very rare to come by.

5. Statement with reasons whether the statement *q*  is a negation of the statement *P* in each of the following:

(a) *p*. The line AB is parallel to the line CD,

*q.* The line AB is perpendicular to the line CD.

(b) *p*. He is a good leader.

*q.* He is a bad leader.

(c) *p*. She is a good leader.

*q.* She is a good follower.

(6) Write the negation of each of the following avoiding the word ‘not’ as much as possible.

(a) The car is moving fast.

(b) He was present in school yesterday.

(c)The Equator is a Great circle.

(d) His friend is younger than my brother.

(7) Write the negation of each of the following avoiding the word ‘not’ as much as possible.

(a) He obtained the least mark in the examination.

(b) She is the shortest girl in the class.

(c) He is an ugly man.

(d) The hospital is in a bad state.

**Conditional Statement**

Let *p* stand for the statement ‘Lagos is a state in Nigeria’ and *q* stand for the statement ‘Lagos is a state in Africa’. One way the two statements can be combined is ‘If Lagos is a city in Nigeria then Lagos is a city in Africa or ‘if *p* and *q*’.

The statement ‘‘if *p* and *q*’ is a combination of two simple statements *p* and *q* it is therefore called a **compound statement.**

Symbolically, we can write the compound statement if *p* then *q*as *p q*.

The statement*p* *q* is read as:

*p* implies q or

If p then q or

*q* is necessary for *p* or

*p* is sufficient for *q* or

*p* only if *q* or

*p* follows from *p* or

*q*if*p.*

The symbol is an operation. In the compound statement*p* is called the **antecedent** while the sub statement*q* is called the **consequent** of *p q.*

The truth or falsity of the implication *p q* is illustrated in Table 2.

**Table 2**

|  |  |  |
| --- | --- | --- |
| *p* | *Q* | *P q* |
| T  T  F  F | T  F  T  F | T  F  T  T |

The statement *p* *q* is false if the antecedent is true and the consequent is false. The statement *p* *q* is sometimes called a **Conditional Statement.**

Consider the following statements;

(a) If Cairo is in Africa then 8 is an even number.

(b) If Cairo is in Africa then 8 is an odd number.

(c)If Cairo is in Asia then 8 is an even number.

(d) If Cairo is in Asia then 8 is an odd number.

The statements (a) (c) and (d) are all true but the statement (b) is not true for the simple reason that the antecedent is true while the consequent is false. Note that although in statement (d) both the antecedent and consequent are false, yet the whole statement is true.

**Converse Statement**

Let*p* be the statement ‘a triangle is equilateral’ and *q* the statement ‘a triangle is equiangular’. The statement *p* implies *q.*i.e*p q*. The statement also implies p, i.e*q p*. In other words, if a triangle is equiangular then it is equilateral. The statement *q p* is called the **converse of the statement**

*P q.*

**Inverse Statement**

If *p* is the statement ‘a triangle is equilateral’ and *q* is the statement ‘a triangle is equiangular’ the statement ̴*p*  ̴*q* is the statement ‘if a triangle is not equilateral than it is not equiangular.

The statement ̴*p*  ̴*q*is called the **inverse of the statement** *p q.*

**Contrapositive Statement**

If *P* is the statement ‘a triangle is equilateral’ and *q* is the statement ‘a triangle is equiangular’ the statement ̴*q*  ̴*p* is ‘if a triangle is not equiangular then it is not equilateral’. The statement

*q p* is called the **contrapositive statement** of *p q.*

**Biconditional Statement**

Let *p* be the statement ‘the interior angles of a polygon are equal’ and *q* be the statement ‘a polygon is regular’.

*p q* is the statement’If the interior angles of a polygon are equal then the polygon is regular’

*q p* is the statement ‘If a polygon is regular then the interior angles of the polygon are equal’. We see here that*p q* and *q p.*

The two conditional statements are valid. We say that *p* and *q* imply ‘each other or p is equivalent to *q* and we write *p q*. The statement *p q* is called a **Biconditional Statement** of *p and q* and the

statements*p* and *q* are said to be equivalent to each other.

Other terminologies for *p q* are:

*q* is equivalent to *p*

*p* is necessary and sufficient for *q*

*q*if and only if *p*

*p* if and only if *q*

If *p*thenq and if *q* then*p.*

The truth of falsity of *pq* is completely illustrated by Table 3.

**Table 3**

|  |  |  |
| --- | --- | --- |
| *p* | *q* | *P q* |
| T  T  F  F | T  F  T  F | T  F  F  T |

So a biconditional statement is true when the two substatements have the same truth value.

Consider the following four statements:

(a) Nyerere is an African name if and only if a, e, i, o, u are vowels;

(b) Nyerere is an African name if and only if a, e, i, o, u are consonants;

(c) Nyerere is a European name if and only if a, e, i, o, u are vowels;

(d) Nyerere is a European name if and only if a, e, i, o, u are are consonants.

The statements (a) and (d) are both true since the substatements of each have the same truth value. The statements (b) and (c) are false since the substatement of each have different truth values.

**The Chain Rule**

If *p. q* and*r* are three statements such that *p q* and *q r,* then *p r.* This is called the **chain rule**and it may have several links.

Consider the argument:

T1: If a student works very hard, he passes his examination.

T2: If a student passes his examination, he is awarded a certificate.

T1: If a student works very hard, he is awarded a certificate.

Let *P* be the statement ‘a student works very hard.

Let *q* be the statement ‘a student passes his examination.

Let *r* be the statement ‘a student is awarded a certificate.

The argument has the following structural form:

*p p. q r : p r*

This argument follows the chain rule link hence it is said to be valid.

The statement T1 and T2 are called the **Premises**while the statement T3 is called the **Conclusion** of the argument.

The argument is valid not on the basis of the truth or falsity of its premises but on the basis of the structural from which follows the chain rule link.

**Compound Statement**

A statement may consist of two or more simple statements or substatements. Such a statement is called

a**Compound** or **Composite Statement.** For example, the compound statement: ‘John is an intelligent and courageous boy’, consists of the substatements: ‘John is an intelligent boy’ and ‘John is a courageous boy’.

By convention, we shall use letters *p, q, r,…* to denote statements.

We proceed now to introduce logical symbols called **Connectives.**

**Disjunction**

Two statements can be combined by the use of the connective ‘or’.

The statement: ‘He is a philosopher’ can be combined with the statement ‘He is a teacher as follow: ‘He is a philosopher or he is a teacher’. In a more refined English, the combined statement is ‘Either he is a philosopher, or he is a teacher’.

If the statement ‘He is a philosopher’ is denoted *p* and the statement ‘He is a teacher is denoted *q,* the combined statement is either *p* or *q*or simply *q or q.* In symbolic logic, *q* or *q* is designated *p* v *q,* where the connective *p* v *q.*Where the connective v means ‘or’.

The word ‘or; in English Language is used in two different sense.

In the statement, ‘the professor will deliver the lecture at Bayero University or remain at the hotel’. The ‘or’ is used in an exclusive sense, in that it is not possible for the professor to deliver lecture at Bayero University, and at the same time remain at the hotel. The ‘or’ is said to be used in the exclusive sense.

In the statement, ‘He is a good politician or a good statement’, the or is used in the inclusive sense, in that it is possible for the good politician to be a good statesman as well. We shall clear this apparent ambiguity, it we settle for one meaning.

**In this teat, we use ‘or’ in the inclusive sense.** The statement *p v q* can be read; either *p or q* or both or simply *p v q* or both. The truth table for *p v q* is illustrated in Table 4

**Table 4**

|  |  |  |
| --- | --- | --- |
| *p* | *q* | *P v q* |
| T  T  F  F | T  F  T  F | T  T  T  F |

The statement *p v q* is false when both *p* and *q* are false, otherwise *p v q* is true.

Consider the following statements:

(a) Cairo is in Africa or 8 is an even number.

(b) Cairo is in Africa or 8 is an odd number.

(c) Cairo is in Asia or 8 is an even number.

(d) Cairo is in Asia or 8 is an odd number.

The statements (a) (b) and (c) are all true since at least one of the substatements is true. The statement (d) is false since the two substatements are false.

**Conjunction**

Another way by which two statements can be combined with the statement ‘she is proud’ using the connective ‘and’ as ‘she is pretty and she is proud’. In a more refined English, the combined statement is: ‘she is pretty and proud’.

If the statement ‘she is pretty’ is denoted *p* and the statement ‘she is proud’ is denoted *q,* the statement ‘she is pretty and proud’ can be written as *p* and *q.* In symbolic logic, *p* and *q* is designated *p ᴧq,* where

the connective *ᴧ* means ‘and’.

The truth table for *p ᴧ q* is illustrated in Table 5

**Table 5**

|  |  |  |
| --- | --- | --- |
| *p* | *q* | *P ᴧ q* |
| T  T  F  F | T  F  T  F | T  F  F  F |

***The statementp ᴧ q* is true when the sub statements *p* and *q* are both true, otherwise *p ᴧ q* is false.**

Consider the following statements:

(a) Cairo is in Africa and 8 is an even number.

(b) Cairo is in Africa and 8 is an odd number.

(c) Cairo is in Asia and 8 is an even number.

(d) Cairo is in Asia and 8 is an odd number.

The statements (a) (b) and (c) are all true since at least one of the substatements is true. The statement (d) is false since the two substatements are false

Of the four given statements, only (a) is true. The statements (b), (c) and (d) are since at least one if their substatement is false.

**Equivalent Statements**

**Two compound statements are said to be logically equivalent if they have the same truth value.**

Use the truth table to establish that:

̴(*p v q)* = ̴*p* ᴧ ̴*q*

**Solution**

**Table 6**

|  |  |  |  |
| --- | --- | --- | --- |
| *p* | *Q* | *p ᴧ q* | ̴(p v q) |
| T  T  F  F | T  F  T  F | F  F  F  T | F  F  F  T |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *P* | *q* | *̴p* | ̴*q* | ̴pᴧ ̴q |
| T  T  F  F | T  F  T  F | F  F  T  T | F  F  F  T | F  F  F  T |

(a) (b)

We observe that the last columns of Table 6(a) and Table 6(b) have the same truth values.

Hence ̴(*p v q)* = ̴*p ᴧ* ̴*q*

Use the truth table to prove that:

̴(*p ᴧ q)* = ̴*p v* ̴*q*

**Solution**

**Table 7**

|  |  |  |  |
| --- | --- | --- | --- |
| *p* | *q* | *p ᴧ q* | ̴(p v q) |
| T  T  F  F | T  F  T  F | T  F  F  F | F  T  T  T |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *P* | *Q* | *̴p* | ̴*q* | ̴pᴧ ̴q |
| T  T  F  F | T  F  T  F | F  F  T  T | F  T  F  T | F  T  T  T |

(a) (b)

The last columns of the two tables have the same truth values, hence ̴(*p ᴧ q)* = ̴*p v*  ̴*q*

Use the truth table to show that:

(a) the connective ᴧ distributes over the connective v;

(b) the connective v distributes over the connective ᴧ.

**Solution**

Given the statements *p, q* and *r* and the connectives ᴧ and v, we wish specifically to show that:

(a) p ᴧ (q v r) = (p ᴧ q) v (p ᴧ r)

(b)p v (q ᴧ r) = (p v q) ᴧ (p v r)

(a)

**Table 8**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *p* | *Q* | *R* | q v r | pᴧ (q v r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  T  F | T  T  T  F  F  T  T  T | T  T  T  F  F  F  F  F |

**Table 9**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *p* | *q* | *r* | p ᴧ q | p ᴧ r | (p ᴧ q) v (p ᴧ r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  T  T  F  F  T  T  T | T  F  T  F  F  F  F  F | T  T  T  F  F  F  F  F |

We notices that the last two columns of the two tables above the same truth values,

Hence p ᴧ (q v r) = (p ᴧ q) v (p ᴧ r)

(b)

**Table 10**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *p* | *q* | *R* | q ᴧ r | p v (q ᴧ r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  F  F  F  F  F  F  T | T  T  T  T  F  F  F  T |

**Table 11**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *p* | *q* | *r* | p v q | p v r | (p v q) ᴧ (p v r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  T  T  T  F  F  T  T | T  T  T  T  F  T  F  T | T  T  T  T  F  F  F  T |

We see here also that the last two columns of Table 10 and Table 11 have the same truth value, hence

p v (q ᴧ r) = (p v q) ᴧ (p v r)

Given the statements p, q and r show that:

(a) p ᴧ (q ᴧ r) = (p ᴧ q) ᴧ r

(b) p v (q v r) = (p v q) v r

**Table 12**

(a

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *p* | *q* | *R* | q ᴧ r | p v (q ᴧ r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  F  F  F  F  F  F  T | T  F  F  F  F  F  F  F |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *P* | *Q* | *r* | p ᴧ q | (p ᴧ q) ᴧ r |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  T  F  F  F  F  F  F | T  F  F  F  F  F  F  F |

(a) (b)

The last two columns of Table 12(a) and Table 12(b) are identical, hence;

p ᴧ (q ᴧ r) = (p ᴧ q) ᴧ r

We see here that the conjunctive connective as an operator, is associative.

(b)  **Table 13**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *P* | *Q* | *r* | p ᴧ q | (p v q) v r |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  F  F  F  F  F  F  T | T  T  T  T  F  T  T  T |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *p* | *q* | *r* | q v r | p v (q v r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  T  T  F  F  T  T  T | T  T  T  T  F  T  T  T |

Table last two columns of Table 13a and Table 13b are identical, hence p v (q v r) = (p v q) v r

**The disjunctive connective as an operator is seen as being associative.**

**Tautology and Contradiction**

A compound statement which is always true irrespective of the truth values of the substatements, is called a **Tautology**. A tautology is represented as **T.**

A compound statement which is always false, irrespective of the truth values of the substatements is called a **Contradiction.** A contradiction is usually represented as **F.**

Ues the truth table to show that the statement p v ̴p is a tautology.

**Solution**

**Table 14**

|  |  |  |
| --- | --- | --- |
| p | ̴p | p v ̴p |
| T  T  F  F | F  F  T  T | T  T  T  T |

We observe that the last column of the table has the truth value T irrespective of the truth values of the substatements. Hence the statement p v ̴p is a tautology.

Use the truth table to show the statement p ᴧ ̴p is a contradiction.

**Solution**

**Table 15**

|  |  |  |
| --- | --- | --- |
| p | ̴p | p ᴧ ̴p |
| T  T  F  F | F  F  T  T | F  F  F  F |

The last column of Table 15 has the truth value F irrespective of the truth values of the substatements hence, p ᴧ ̴p = F.

Show that the statement

p ᴧ (( ̴p ᴧ q) v ( ̴p ᴧq)) is a contradiction.

**Table 16**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | Q | ̴p | ̴q | ̴p ᴧ q | ̴p ᴧ ̴q | ( ̴p ᴧ q) v  ( ̴p ᴧ ̴q) | p ᴧ (( ̴p ᴧ q) v  ( ̴p ᴧ ̴q)) |
| T  T  F  F | T  F  T  F | F  F  T  T | F  T  F  T | F  F  T  F | F  F  F  T | F  F  T  T | F  F  F  F |

We see that the truth value of expression in the last column of Table 16 is F irrespective of the truth values of the substatements, hence p ᴧ (( ̴p ᴧ q) v( ̴p ᴧ ̴q)) = F

**Laws of the Algebra of Logical Statements**

There is a close relationship between the algebra of sets and algebra of logical statements. The logical connectives as operations obey the laws of algebra.

**Commutative Laws**

i. (a) p ᴧ q = q ᴧ p

I (b) p v q = q v p

**Associative Laws**

Ii (a) p ᴧ (q ᴧ r) = (p ᴧ q) ᴧ r

Ii (b) p v (q v r) = (p v q ) v r

**Distributive Laws**

iii. (a) p ᴧ (q ᴧ r) = (p ᴧ q) ᴧ (p ᴧ r)

iii. (b) p v (q v r) = (p v q ) v (p v r)

**Laws of Absorption**

iv. (a) p ᴧ (p v q) = p

iv. (b) p v (p ᴧ q) = p

**Idempotent Laws**

v. (a) p ᴧ p = p

v. (b) p v p = p

**De Morgan’s Laws**

vi. (a) ̴(p ᴧ q) = ̴p c ̴q

vi. (b) ̴(p v q) = ̴p ᴧ ̴q

**Laws of Complementation**

vii. (a) p ᴧ ̴p = F (b) p v ̴p = T

vii. (a) ̴F = T (b) ̴T = F

ix. ̴( ̴p) = P

**Laws of Contrapositivity**

x. p q = ̴q ̴p

**Laws of Identity**

xi. P ᴧ T = P

xii. P ᴧ F = F

xiii. P v T = T

xiv. P v F = P

All these laws can be verified using the truth table technique.

Using the truth table technique, show that if p, q and r are arbitrary statements, then:

(p q) ᴧ (q r) (p r) = T

**Solution**

**Table 17**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p q | q r | (p q)r (q r) | p r | (p q) ᴧ  (q r)  (p r) |
| T  T  T  T  F  F  F  F | T  T  F  F  F  F  T  T | T  F  T  F  F  T  F  T | T  T  F  F  T  T  T  T | T  F  F  F  T  T  F  T | T  F  F  F  T  T  F  T | T  F  T  F  T  T  T  T | T  T  T  T  T  T  T  T |

The truth value of the expression in the last column is T, hence

(p q) ᴧ (q r) (p r) = T

Use the truth table technique to show that

p q = (p q) ᴧ (q p)

**Solution**

**Table 18**

(a)

|  |  |  |
| --- | --- | --- |
| p | Q | p q |
| T  T  F  F | T  F  T  F | T  F  F  T |

(b)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | Q | p q | q p | (p q) ᴧ (q p) |
| T  T  F  F | T  F  T  F | T  F  T  T | T  T  F  T | T  F  F  T |

The truth values of the expression in the last columns of each of the table are identical, hence

p q = (p q) ᴧ (q p)

Use the truth table technique to show that

̴p ̴q = q p

**Solution**

**Table 19**

p q ̴p ̴q ̴p ̴q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| T | T | F | F | T |
| T  F  F | T  F  T | F  T  T | T  F  T | T  F  T |

p q q p

|  |  |  |
| --- | --- | --- |
| T | T | T |
| T  F  F | F  T  F | T  F  T |

(a) (b)

The last column of the two tables are identical, hence

̴p ̴q = q p

1. Let *P* be the statement ‘He is funny’ and *q* be the statement ‘He is serious’. Write each of the following in a simple English:

(a) p v q (b) p ᴧ q

(c) p ᴧ ̴q (d) ̴p v ̴q

2. Let *p* be the statement ‘she is beautiful’ and *q* be the statement ‘she is soft - spoken’. Write each of the following in symbolic form:

(a) She is beautiful and soft – spoken.

(b) Either she is beautiful or she is soft – spoken.

(c) She is beautifulbut not soft – spoken.

(d) She is ugly but soft – spoken.

**Evaluation**

1. Fin the truth value of these statements (a) If 11 > 8 then -1 < -8(b) If 3 + 4 ≠ 10 then 2 + 3 ≠ 5.

**General Evaluation**

(1) Let P be the statement: “He is funny” and q be the statement: “He is serious”. Write each of the following in simple English (i) p v q (b) p ᴧ ̴q (c) ̴p v ̴q

(2) If p and q represent two statements ‘he is good in physics” and “he is good in mathematics” respectively. Write the following in symbolic form; “he is good in physics if and only if he is good in mathematrics”.

**Reading Assignment**

F/Maths Project 1 pages 126 – 130 Exercise 9b Q 2, 3 and 4

**Weekend Assignment**

**Objective**

P is the statement “Ayo has determination and q is the statement”Ayo will succeed”. Use this information to answer the questions. Which of these symbols represent these statements?

(1) Ayo has no determination (a) p q (b) ̴p q (c) ̴p

(2) If Ayo has no determination then he won’t succeed (a) ̴p ̴q (b) p ̴q (c) p q

(d) p ̴q

(3) If Ayo won’t succeed then he has no determination (a) ̴q p (b) ̴q ̴q (c) ̴q p

(d) q p

(4) If Ayo has determination then he will succeed (a) ̴p q (b) ̴p ̴q (c) ̴q ̴p

(d) p q

(5) If Ayo has no determination then he will succeed (a) ̴p q (b) ̴q ̴p (c) ̴p

(d) ̴p ̴q

**Theory**

(1) Write down the inverse, converse and contrapositive of each of these statements.

(i) If the bank workers work hard they will be adequately compensated.

(ii) If he is humble and prayerful, he will meet with God’sfavour.

(iii) If he set a good example, he will get a good followership.

(2) Consider the following statements P: some dogs are tame Q: all tame animals are small.

Which of the following is a valid conclusion from the above statements?

(i) All dogs are tame (ii) No dog is small (iii) All small animals are tame (iv) Some dogs are small (v) All tame animals are dogs.

**WEEK 8**

**TOPIC: Trigonometric functions**

The basic trigonometric ratios can be defined in two ways:

(i)traditional definition;

(ii) modern definition.

**Traditional Definition**

The basic trigonometric ratios can be defined in terms of the sides of a right – angled triangle.

Q

r p

P ѳ

q R

**Fig. 14.3**

∆PQR in Fig 14.3 in a right angled triangle with QPR = ѳ and PRQ = 90◦. We define the three basic rations as follows:

Cosine of angle ѳ =

Sine of angle ѳ =

Tangent of ѳ =

The cosine of angle ѳ, sine of angle ѳ and the tangent of angle ѳ will be abbreviated cosѳ, sinѳ and tanѳ respectively.

Thus:

Cosѳ =

Sinѳ =

Tanѳ –

Also:

=

tanѳ =

**Ratios of the General Angle**

**First Quadrant**

ѳ

P1

y

**Fig 14.7**

In Fig. 14.7 ∆OP1N1 is a right – angled triangle constructed from a unit circle.

OP1 = 1

P1N1 = y

ON1 = x

P1ON1=ѳ1

Sinѳ1 = y

Cos̴ѳ1 = x

Tanѳ1 =

**Second Quadrant**

P2

1

y

(a)

y

P2 P1

Ѳ1 ѳ2 ѳ1

x

(b)

**Fig. 14.8**

From Fig. 14.8(a)

Sinѳ2 = y

Cosѳ2 = -x

Tanѳ2 =

=

From Fig. 14.8(b)

Ѳ1 + ѳ2 = 180◦

Ѳ2 = 180◦ - ѳ1

: Sinѳ2 = sin(180◦ - ѳ1) = y = sinѳ1

: Sin(180◦ - ѳ) = sinѳ

: Cosѳ2 = cos(180◦ - ѳ1) = -x = -cosѳ1

:Cos(180◦ - ѳ) = -cosѳ

**Similarly,**

Tanѳ2 = tan (180◦ - ѳ1) = 1

: tan (180◦ - ѳ) = -tanѳ

Hence in the second quadrant:

Sin(180◦ - ѳ) = sinѳ

Cos(180◦ - ѳ) = -cosѳ

Tan(180◦ - ѳ) = -tanѳ

**Third Quadrant**

N1– x ѳ3 (a)

-y Q

P3 1

y

P1

(b) ѳ1

Ѳ1

Ѳ1

P3

**Fig. 14.9**

Sinѳ3 = -y

Cosѳ3 = -x

Tanѳ3 = sinѲ3 =

cosѲ3

From Fig. 14.9(b)

Ѳ1 = 180◦ + Ѳ1

SInѳ3 = sin(180◦ + Ѳ) = -y = -sinѲ1

:sin(180◦ + Ѳ) = -sinѲ

Cosѳ3 = cos(180◦ + Ѳ) = -x = -cosѲ1

: cos(180◦ + Ѳ) = -sinѲ

Similarly,

Tanѳ3 = tan (180◦ + Ѳ1)= = tanѳ1

: tan(180◦ + ѳ) = tanѳ

Hence in the third quadrant:

Sin(180◦ + ѳ) = -sinѳ

Cos(180◦ + ѳ) = -cosѳ

Tan(180◦ + ѳ) = tanѳ

**Fourth Quadrant**

(a)

x

-y

Ѳ1 1

P4

y

(b) P1

Ѳ1

Ѳ1 x

Ѳ1

P2

**Fig. 14.10**

Sinѳ4 = y

Cosѳ4 = x

Tanѳ4 =

From Fig, 14.10(b)

Ѳ4 + Ѳ1= 360◦

Ѳ4 = 360◦ - ѳ

SInѳ1 = sin(360◦ + Ѳ) = -y = -sinѲ1

: sin(360◦ + Ѳ) = -sinѲ

Cosѳ1 = cos(360◦ + Ѳ) = -x = -cosѲ1

: cos(360◦ + Ѳ) = -sinѲ

Tanѳ4 = tan (360◦ + Ѳ)= = -tanѳ

: tan(360◦ + ѳ) = -tanѳ

Hence in the third quadrant:

Sin(360◦ + ѳ) = -sinѳ

Cos(360◦ + ѳ) = cosѳ

Tan(360◦ + ѳ) = -tanѳ

(a) In the first quadrant, all the ratios are positive.

(b) In the second quadrant, only sin ratio is positive, while the rest are negative.

(c) In the third quadrant, only tangent ratio is positive, while the rest are negative.

(d) In the fourth quadrant, only cosine ratio is positive, while the rest are negative.

These observations can be summarized in the figure below:

y y

SINL ALL S A

T C

(a) (b)

**Negative Angles**

y

Ѳ -y

360◦ - 0

x

P

Ps

(a) (b)

Fig. 14.12

Since negative angles are measured in the clockwise sense, the direction of OP when rotated through –ѳ is the same as where it is rotated through 360◦ - ѳ.

Hence in the forth quadrant:

Sin(-ѳ) sin sin(360◦ - ѳ) = -sinѳ

Cos(-ѳ) cos(360◦ - ѳ) = -cosѳ

Tan(-ѳ) tan(360◦ - ѳ) = -tanѳ

**Use tables to evaluate each of the following:**

(a) sin 143◦ (b) cos 115◦

(c) tan 125◦

**Solution**

(a) 143◦ is in the second quadrant, so

Sin143◦ = sin(180◦ - 143◦)

= sin37◦

= 0.6018

(b) 115◦ is in the second quadrant, so

Cos115◦ = -cos(180◦ - 115◦)

= -cos65◦

= -0.4226

(c) 125◦ is in the second quadrant, so

Tan125◦ = -tan(180◦ - 125◦)

= -tan55◦

= -1.428

Use tables to evaluate each of the following

(a) sin230◦ (b) cos236◦

(c)tan 242◦

**Solution**

220◦, 236◦ and 242◦ are all in the third quadrant, hence;

(a) sin220◦ = sin(180◦ + 40◦)

= -sin40◦

= -0.6428

(b) cos236◦ = cos(180◦ + 56◦)

= -cos56◦

= -0.5992

(c) tan242◦ = tan(180◦ + 62◦)

= tan62◦

= 1.881

Use tables to evaluate each of the following:

(a) sin310◦ (b) cos285◦

(c) 334◦

**Solution**

310◦, 285◦ and 334◦ are all in the fourth quadrant, hence;

(a) sin310◦ = sin(360◦ - 50◦)

= -sin50◦

= -0.7660

(b) cos285◦ = cos(360◦ - 75◦)

= cos75◦

= 0.2588

(c) tan334◦ = tan(360◦ - 26◦)

= -tan26◦

= -0.4877

Use tables to evaluate each of the following

(a) cos(-30◦) (b) sin(-60◦)

(c) tan(-120◦)

**Solution**

(a) cos(-30◦) = cos330◦

= cos30◦

= 0.8660

(b) sin(-60◦) = sin300◦

= -sin60◦

= -8660

(c) tan(-120◦) = tan240◦

= tan60◦

= 1.732

Use the table to find the value of ѳ between ѳ◦ and 360◦ which satisfy each of the following:

(a) cosѳ = -0.4540

(b) tanѳ = 1.176

(c) sinѳ = -0.9336

**Solution**

(a) The cosine ratio is negative in the second and third quadrants. First find the acute angle whose cosine is 0.4540

From the tables cos 63◦ = 0.4540

: In the second quadrant

Ѳ = 180◦ - 63◦

= 117◦

In the third quadrant,

Ѳ = 180◦ + 63◦

= 243◦

(b) The tangent ratio is positive in the first and third quadrants.

First find the acute angle whose tangent is 1.176.

From the tables.

Tan49.62◦= 1.176◦

In the first quadrant.

Ѳ = 49.62◦

In the third quadrant.

Ѳ = 180◦+ 49.62◦

= 229.62◦

(c) The sine ratio is negative in the third and fourth quadrant.

First find the acute angles whose sine ratio is 0.9336.

From tables.

Sin69◦ = 0.9336

In the third quadrant

Ѳ = 180◦ + 69◦

= 249◦

In the fourth quadrant.

Ѳ = 360◦ - 69◦

= 291◦

**Evaluation**

1. In what quadrant are the followings ; tan ( -540) , cos (- 1080)

**General Evaluation**

(1) Prove that (1 –sinѳ) (1 + sinѳ) = cot2ѳ

sin2ѳ

(2) Show that (secѳ – tanѳ) (secѳ + tanѳ) = 1

(3) Find the values of sin (-210) in surd form

**Reading Assignment**

F/Maths Project 1 pages 225 – 247 Exercise 14 Q1, 3, 5, 7 and 8

**Weekend Assignment**

Given that sinѳ =

(1)Find cosѳ (a)

(2) Find tanѳ

(3) Find cosecѳ

(4) Find secѳ (a)

(5) Find cotѳ

**THEORY**

1) Prove that 1/1+cosx + 1/1-cosx = 2 cosec2 x

2) Given that sin x = 5/13 and x is acute find cosec x

, cot x and sec x

**WEEK 9**

**TOPIC: Graphs of Trigonometric Function**

The graph of the followings will be considered

(a) y = sinѳ, 0◦ ≤ ѳ ≤ 360◦

(b) y = cosѳ, 0◦ ≤ ѳ ≤ 360◦

(c) y = tanѳ, 0◦ ≤ ѳ ≤ 360◦

The graph of y = sinѳ, ѳ◦ ≤ ѳ ≤ 360◦

On the graph sheet, draw a long horizontal axis in the middle. Mark a point 0’, 3cm to the left of the origin 0. With centre 0’ draw a circle of radius 2cm. Draw a vertical axis through 0. Call the horizontal axis ѳ – axis and the vertical axis y – axis.

On the ѳ – axis choose a scale of 2cm to 1 unit. Using your protractor, mark the angles: 0◦, 30◦, 60◦, 90◦, 120◦, 150◦ … 330◦ as shown in

Draw a horizontal line through 30◦ on the circle. Draw a vertical line through 30◦ on the circle. Draw a vertical line through 30◦ on the ѳ – axis to meet the horizontal line. Mark the point of intersection of these two lines with a small neat cross. Repeat the above procedure for the angles 60◦, 90◦, 120◦, … 330◦. You will obtain a series of points. Join the points by a smooth curve. The curve you obtain is the graph of y = sinѳ.

Essential features of the graph of y = sinѳ:

(a) The graph of y = sinѳ forms a wave – like pattern. It is said to **oscillate.**

(b) The maximum value of y = sinѳ is 1 and it occurs when ѳ = 90◦.

(c) The minimum value of y = sinѳ is -1 it occurs when ѳ = 270◦.

(d) The graph repeats itself at intervals of 360◦. The sine function is an example of a periodic function because it repeats itself at intervals of 360◦. The function is said to have a **periodicity** of 360◦.

(e) The length *AE* on the graph is called the **amplitude** of the function.

**The Graph of y = cosѳ, ѳ◦ ≤ ѳ ≤ 360◦**

The graph of y = cosѳ can be drawn in a manner similar to that of y = sinѳ except that the angles are measured from ***OR*** in the clockwise sense as shown in Fig. 14.17. This is so because cosѳ = sin(90◦ - ѳ).

Essential features of the graph of y = cosѳ:

(a) All the essential features that hold for the graph of y = sinѳ also hold for the graph of y = cosѳ.

(b) In addition, the cosine curve lags behind the sine curve by a difference of 90◦. The difference is usually called a **Phase difference.** In other words, the cosine curve lags behind the sine curve by a phase difference of 90◦.

(c) Both the sine curve and the cosine curve demonstrate some physical phenomena like tidal waves, sound waves alternating currents, e.t.c.

**The Graph of y = tanѳ, ѳ◦ ≤ 360◦**

The graph of y = tanѳ is easier to draw using a table of values than using projections from a unit circle. Make a table of values of y = tanѳ from 0◦ to 360◦ as shown in Table 14.2

Essential features of the tangent curve:

(a) The curve consists of three parts between 0◦ and 360◦.

(b) Since the tangent function is not defined at 90◦ and 270◦, the function is said to be **discontinuous** at these points.(c) The curve rises and falls rapidly at anglesvery close to 90◦ and 270◦ respectively. The curve approaches the vertical lines at 90◦ and 270◦ but never touches them. These vertical lines at 90◦ and 270◦ ate called **Asymptotes**. The asymptotes are shown by dotted lines.

(d) The tangent function is also a periodic function. It has a periodicity of 180◦.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0◦ | 30◦ | 60◦ | 75◦ | 105◦ | 120◦ | 150◦ | 180◦ | 210◦ | 240◦ | 255◦ | 285◦ | 315◦ | 330◦ | 300◦ | 360◦ |
| y = tanѳ | 0 | 0.58 | 1.73 | 3.73 | -3.73 | -1.73 | -0.58 | 0 | 0.58 | 1.73 | 3.73 | -3.73 | -1 | 0.58 | -1.73 | 0 |

Take a scale of 1cm to represent 30◦ on the ѳ- axis and 1cm to represent 1 unit on the y – axis.

**Example**

Using the same axis, a scale of 1cm to represent 30◦ on the ѳ axis and 2cm to represent 1 unit on the y-axis, draw the graphs of the following relations.

(a) y = sinѳ

(b) y = 2sinѳ

(c)y = ½ sinѳ in the interval 0◦ ≤ ѳ ≤ 360◦.

**Solution**

(Refer to table 14.3 and Fig. 14.19)

We observe that the curves:

y = sinѳ

y = 2sinѳ

y = ½ sinѳ

have the same periodicity (360◦), but differ in amplitudes.  
The amplitude of y = sinѳ is 1.

The amplitude of y = 2sinѳ is 2.

The amplitude of y = ½ sinѳis ½.

In general, the curve y = *A*sinѳ has amplitude /*A*/ abd a periodicity of 360◦. This property of the same curve is also a characteristic of the cosine curve.

**Evaluation**

1. Prove that sec2ѳ + cosec2ѳ = (tanѳ + cotѳ) 2.

**General Evaluation**

(1) Draw the graph of y = 2cosx – 1 in the range 0◦ ≤ x ≤ 360◦ at intervals of 30◦.

(2) Draw the graph of y = 3sin x – 1 in the range of 0◦ ≤ x ≤ 360◦ at intervals of 30◦

(3) Sketch the graph of:

(i) y = sin2x (ii) y = cosx

(iii) y = sec x (iv) cosec x

all at intervals of 30◦ range 0≤ x ≤ 360.

**Weekend Assignment**

Given that 4cos x + 3sin x = 5, find the value of

(1) Sin x

(2) Cos x

(3) Tan ѳ

(4) Cot ѳ

(5) Sin x + cos x

**Theory**

(1) Draw the graph of inverse trig function for sin x –

(2) Find the inverse of the following and their domains

(a) y = sin x (b) y = cos x (c) y = tan x

(d) y = cosec x (e) y = sec x (e) y = cot x

**WEEK 10**

**TOPIC: Trigonometric Identities and graphs of inverse trigonometric ratios**

**Pythagoras Theorem**

Ѳ

P

x

**Fig. 10 a**

Fig 10a shows a unit circle ∆OPN is a right – angled triangle with OP = 1, ON = x and PN = y, PON – ѳ. Find the definition of trigonometric ratios.

x = cosѳ …(1)

y = sinѳ …(2)

From (1) x2 = cos2ѳ …(3)

From (2) y2 = sin2ѳ …(4)

Adding (3) and (4)

x2 + y2 = cosѳ2 + sinѳ2 …(5)

Since ∆OPN is a right – angled triangle

ON2 + NP2 = OP2

x2 + y2 = 1 …(6)

Equating (5) and (6)

cos2ѳ+ sin2ѳ = 1 …(7)

Dividing both sides of (7) by cos2ѳ

cos2ѳ + sin2ѳ = 1

cos2ѳ cos2ѳ cos2ѳ

: 1 + tan2ѳ = sec2ѳ

Dividing (7) through by sin2ѳ

Cos2ѳ + 1 = cosec2ѳ

: 1 + cot2ѳ = cosec2ѳ

**Example**

**Show that (3 – sin2ѳ) cosec2ѳ = 2cosec2ѳ + cot2ѳ**

**Solution**

LHS = 3cose2ѳ – sin2ѳ cosec2ѳ

= 3cosec2ѳ – sin2ѳ

Sin2ѳ

= 3cosec2ѳ – 1

= 3cosec2ѳ – (cosec2ѳ- cot2ѳ)

= 3cosec2ѳ – cosec2ѳ + cot2ѳ

= 2cosec2ѳ + cot2ѳ

= RHS

**Example**

Prove that 2cosec2ѳ

**Solution**

LHS =

= 2

1 – cos2ѳ

LHS = 2

sins2ѳ

= 2cosec2ѳ

= RHs

**Example**

Show that = 4cot2ѳ + 2

**Solution**

LHS

= (1 + cosѳ)2 + (1 – cosѳ)2

(1 – cosѳ)(1 + cosѳ)

= 1 + 2cosѳ + cos2ѳ + 1 – 2cosѳ + cos2ѳ

1 – cos2ѳ

= 2cos2ѳ + 2

Sin2ѳ

= 2(cos2ѳ + 1)

Sin2ѳ

= 2(cot2ѳ + cosec2ѳ)

= 2(cot2ѳ + 1 + cot2ѳ)

= 2(2cot2ѳ + 1)

= 4cot2ѳ + 2

= RHS

**Example**

If asinѳ + bcosѳ = p

andacosѳ – bsinѳ = q

show that a2 + b2 = p2 + q2

**Solution**

asinѳ + bcosѳ = p …(1)

acosѳ – bsinѳ = q …(2)

Squaring both sides of (1)

(asinѳ + bcos)2 = p2

: a2 sin2ѳ + 2absinѳ cosѳ + b2cos2ѳ = p2 …(3)

Squaring both sides of (2)

(acosѳ – bsinѳ)2 = q2

a2cos2ѳ – 2absinѳ cosѳ + b2cos2ѳ = q2 …(4)

Adding (3) and (4)

a2sin2ѳ + 2absinѳ cosѳ + b2cos2ѳ + a2cosѳ – 2absinѳ cosѳ + b2sin2ѳ = p2q2

: a2sin2ѳ + a2cos2ѳ + b2cos2ѳ + b2sin2ѳ = p2 + q2

a2(sin2ѳ + cos2ѳ) + b2(cos2ѳ + sin2ѳ) = p2 + q2

But sin2ѳ + cos2ѳ = 1

: a2 + b2 = p2 + q2

**Trigonometric Equations**

A trigonometric equation is an equation containing trigonometric ratios.

Since trigonometric functions are periodic functions, there are infinite numbers of solutions for trigonometric equations. It is therefore the tradition to find some solutions within a given range.

If asinѳ + = 0(a ≠ 0)

then, asinѳ = -b

Ѳ = sin1

Sin1means the angles whose sine ratio is

It is called sine inverse of it is also denoted as sin thus:

Sinѳ =

Ѳ = sin-1

= arc sin

Similarly, if acosѳ + b = 0, (a ≠ 0)

then cosѳ =

ѳ = cos-1

**Evaluation**

**Sketch the graph of:**

(i) y = sin2x (ii) y = cos x

(iii) y = sec x (iv) cosec x

all at intervals of 30◦ range 0≤ x ≤ 360.

**General Evaluation**

(1) Draw the graph of y = 2cosx – 1 in the range 0◦ ≤ x ≤ 360◦ at intervals of 30◦.

(2) Draw the graph of y = 3sin x – 1 in the range of 0◦ ≤ x ≤ 360◦ at intervals of 30◦

(3) Prove that sec2ѳ + cosec2ѳ = (tanѳ + cotѳ) 2.

**Weekend Assignment**

Given that 4cos x + 3sin x = 5, find the value of

(1) Sin x

(2) Cos x

(3) Tan ѳ

(4) Cot ѳ

(5) Sin x + cos x

**Theory**

(1) Draw the graph of inverse trig function for sin x –

(2) Find the inverse of the following and their domains

(a) y = sin x (b) y = cos x (c) y = tan x

(d) y = cosec x (e) y = sec x (e) y = cot x